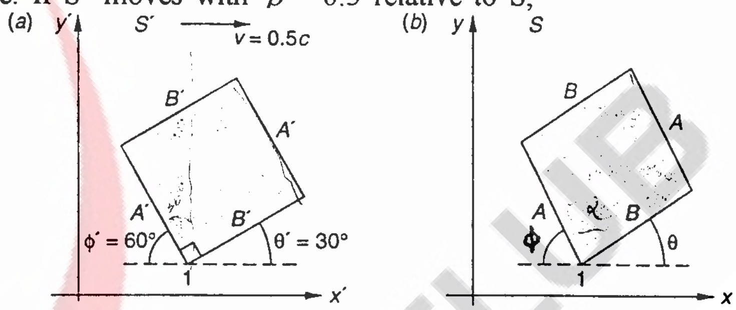
Notre Dame University
Course PHS 213 (Modern Physics)
Academic year 2002-2003
Fall semester
(Instructor: Dr. Youssef K. El-Hage)

November 16, 2002

Quiz #1
(Duration: 1 h)

1) Consider the square in the x'y' plane of S' (A'=B') with one side making a 30^0 angle with the x' axis as in the figure. If S' moves with $\beta = 0.5$ relative to S, determine
   a) side A in terms of A';
   b) side B in terms of B';
   c) the orientation and shape of the figure in S, that is, the angles θ and ϕ and the interior angle at vertex 1. Is the figure in S still a square?



(30 pts.)

2) An electron (rest energy = 0.511 MeV) and a proton (rest energy = 938.3 MeV) are each accelerated through an electric potential of 10 MV.
   a) Find γ, the momentum p and the speed for each.
   b) For the electron case, what is the error done if p is calculated by means of the extreme relativistic (or high-energy) approximation?
   c) For the proton case, what is the error done if p is calculated by means of the nonrelativistic approximation?

(30pts.)

3) The threshold wavelength of potassium is 558 nm. What is the stopping potential when incident light of wavelength 400 nm is used in a photoelectric experiment on potassium?

(15 pts.)

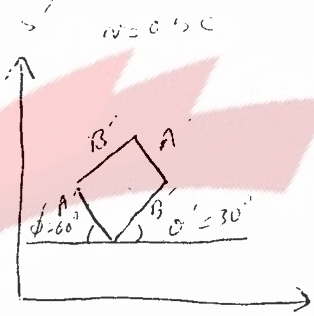
4) In a particular Compton scattering experiment, it is found that the incident wavelength λ_0 is shifted by 1.5 percent when the scattering angle $\theta = 120^0$.
   a) What is the value of the scattered wavelength λ '?
   b) What will be this scattered wavelength when the scattering angle is 75^0?

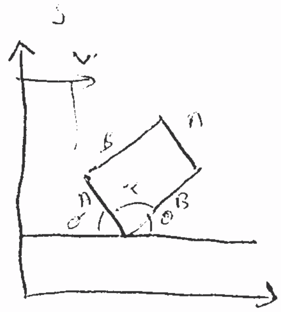
(25 pts.)

GOOD LUCK

- Open-book exam (only the official textbook is allowed).
- Borrowing anything during exam is strictly prohibited.
- The following cheating policy will be applied strictly:
  No fraud, deception, talking, trick, signs, gestures, copying from another student and the unauthorized use of study aids, books, data or other information is allowed. Immediate disciplinary measures will be taken against cheating students (Section 10.2 (b) of the University Constitution).

(1)





a) $A = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{A'_x}{\gamma}\right)^2 + (A'_y)^2}$

$= \sqrt{\left(\frac{A'\cos\theta}{\gamma}\right)^2 + (A'\sin\theta)^2} = A'\sqrt{\left(\frac{\cos 60^\circ}{\gamma}\right)^2 + \sin^2 60}$

$\frac{1}{\gamma} = \sqrt{1-\beta^2}$

$\beta^2 = 0.75 = \frac{3}{4}$

$A = A'\sqrt{\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \frac{3}{4}} = A'\sqrt{\frac{15}{16}}$

$A = 0.968\,A'$

b) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{\left(\frac{B'_x}{\gamma}\right)^2 + (B'_y)^2}$

$= \sqrt{\left(\frac{B'\cos\theta'}{\gamma}\right)^2 + (B'\sin\theta')^2} = B'\sqrt{\left(\frac{\cos 30^\circ}{\gamma}\right)^2 + (\sin 30^\circ)^2}$

$= B'\sqrt{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) + \frac{1}{4}} = B'\sqrt{\frac{13}{16}}$

We know that $A' \perp B'$

$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\frac{B'\sin 30^\circ}{B'\cos 30/\gamma} = \tan^{-1}\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\sqrt{\frac{4}{3}}$

$= 33.7^\circ$

$\phi = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{A'\sin 60^\circ}{A'\cos 60^\circ/\gamma} = 63.4^\circ$

$\alpha = 180^\circ - \theta - \phi = 82.9^\circ = \alpha$

(2)

$m_e c^2 = 0.511$ MeV

$m_p c^2 = 938.3$ MeV

$V = 10$ MV

$K = qV = eV$ for Both $= 10$ MeV

for the e^-

* $\gamma = \frac{E}{m_e c^2} = \frac{K + m_e c^2}{m_e c^2} = 1 + \frac{K}{m_e c^2}$

$= 1 + \frac{10 \text{ MeV}}{0.511 \text{ MeV}} = 20.57 = \gamma$

for proton

$\gamma = \frac{E}{m_p c^2} = 1 + \frac{K}{m_p c^2}$

$\gamma = 1 + \frac{10 \text{ MeV}}{938.3 \text{ MeV}}$

$= 1.01$

* $P = \frac{1}{c}\sqrt{E^2 - (m_e c^2)^2} = \frac{1}{c}\sqrt{(10.511 \text{ MeV})^2 - (0.511 \text{ MeV})^2}$

$= 10.50 \frac{\text{MeV}}{c}$

* $P = \frac{1}{c}\sqrt{E^2 - m_p c^2} = \frac{1}{c}\sqrt{(948.3 \text{ MeV})^2 - (938.3 \text{ MeV})^2}$

$= 137.4 \frac{\text{MeV}}{c}$

for the electron

* $\frac{v}{c} = \frac{Pc}{E} = \frac{10.50 \text{ MeV}}{10.511 \text{ MeV}}$

$= 0.99c$

$v = 0.99c$

proton

$\frac{v}{c} = \frac{Pc}{E} = \frac{137.4 \text{ MeV}}{948.3 \text{ MeV}}$

$v = 0.145c$

In the extreme relativistic case

$$K \gg m_e c^2 \Rightarrow E \gg m_e c^2$$

$$p = \frac{1}{c}\sqrt{E^2 - (m_e c^2)^2} \approx \frac{E}{c} = 10.511 \ \frac{\text{Mev}}{c}$$

$$E_{error} = \frac{10.511 \text{ Mev} - 10.50 \text{ Mev}}{10.50 \text{ Mev}} = +0.095$$

$\simeq 9.5\%$ overestimation

In the nonrelativistic case

$$K = \frac{1}{2} m_p v^2 = \frac{p^2}{2m_p} = \frac{p^2 c^2}{2 m_p c^2}$$

$$p = \frac{1}{c}\sqrt{K(2m_p c^2)} = \frac{1}{c}\sqrt{10\text{MeV}(2 \times 938.3\text{Mev})} = 137.0\text{Mev}$$

$$\text{Error} = \frac{137.0 \text{ Mev} - 137.4 \text{ Mev}}{137.4 \text{ Mev}} \approx 3\% \text{ underestimation}$$

(3) $\lambda_{th} = 558$ nm $\quad \lambda = 500$ nm

$$\nu_{th} = \frac{\phi}{h}$$

$$eV_0 = h\nu - \Phi = h\nu - h\nu_{th} = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_{th}}\right)$$

$$V_0 = \frac{1240 \text{ eV}}{e}\left(\frac{1}{500} - \frac{1}{558}\right) = 0.26\text{V}$$

(4) $\lambda' - \lambda_0 = \lambda_{c}(1 - \cos\alpha)$

$0.013\lambda_0 = 0.00243 \text{nm}(1 - \cos 120^\circ)$